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SOLUTION OF A PROBLEM ON TWO-SIDED HEAT EXCHANGE IN HEAT EXCHANGERS WITH A TWISTED FLOW

V. M. Budov and S. M. Dmitriev

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Analytical relations are obtained for the temperature profile of heat carriers along the flow axis in a heat exchanger with an inner spiralled tube in the case of two-sided heating.

A problem that is presently very important is reducing the weight and dimensions of a heat exchanger widely used in different areas of technology. An effective way of solving this problem is developing compact heat-exchange surfaces.

An example of an apparatus with a high energy intensity is a heat exchanger with two-sided heating and elements in the form of "tube-in-tube" channels with a spiral inner tube, where part of the outside of the spiral tube is in contact with the outer tube along the helix. The heating medium thus moves inside the inner tube and between the tubes, while the heated medium moves in the spiral channel formed between the surfaces of the spiralled inner tube and the outer tube.

Such a design increases the compactness coefficient by a factor of 1.5-1.8 compared to straight-tube heat exchangers with one-sided heating, while the spiralling of the inner tube helps intensify heat transfer both on the side of the heating medium and on the side of the heated medium as a result of twisting of the flow of heat carriers.

Existing methods of calculating the temperature fields of heat exchangers with two-sided heating [1-3] do not take into account heat exchange which occurs between the flows of the heating heat carrier at the site of contact of the spiralled inner tube and the outer tube over the entire length of the element.

This article presents analytical relations which make it possible, given prescribed boundary conditions, to obtain the distribution not only of the outlet temperatures, but also

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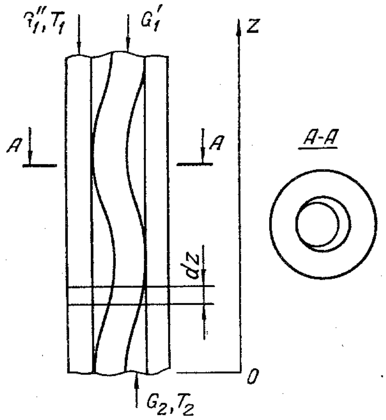

Fig. 1

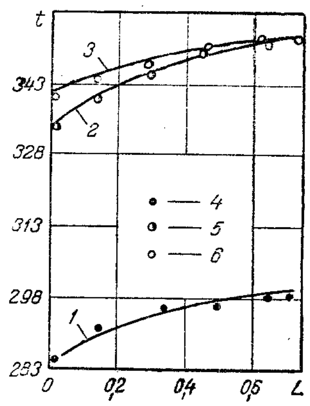

Fig. 2

Fig. 1. Design diagram of heat-exchanger element.

Fig. 2. Distribution of temperatures of the heat carriers along the heat exchanger: 1, 2, 3) theoretical values for the heated medium, the heating medium in the inner tube, and the heating medium in the space between the tubes, respectively; 4, 5, 6) experimental values for the corresponding heat carriers. t, °K; L, m.

of the temperatures of the heating and heated heat carriers along the flow axis. In solving the problem, we assume that the heat-transfer coefficients and specific heats of the heat carriers are constant along the heat exchanger. Figure 1 shows a diagram of an element of the apparatus.

The heat-balance equations on the section dz of the heat exchanger are represented by a system of homogeneous first-order differential equations with constant coefficients:

$$\begin{aligned} W_1' \frac{dt_1'}{dz} &= k_1(t_1' - t_2) - k_3(t_1'' - t_1'), \\ W_2 \frac{dt_2}{dz} &= k_1(t_1' - t_2) + k_2(t_1'' - t_2), \\ W_1'' \frac{dt_1''}{dz} &= k_2(t_1'' - t_2) + k_3(t_1'' - t_1'). \end{aligned} \tag{1}$$

Here, $k_1 = f_1K_1$, $k_2 = f_2K_2$ and $k_3 = (1 - f_3)K_3$, while the percentages of free surface participating in convective heat exchange between the heating and heated media f_1, f_2, and f_3 are determined by the expressions

$$f_1 = 1 - \frac{p_k}{\pi d_2}, \quad f_2 = 1 - \frac{p_k}{\pi d_3}, \quad f_3 = \frac{2f_1f_2}{f_1 + f_2}.$$

We find the linear heat-transfer coefficients K_1, K_2, and K_3 from the familiar relations

$$K_1 = \pi / \left(\frac{1}{\alpha_1 d_1} + \frac{1}{2\lambda_1} \ln \frac{d_2}{d_1} + \frac{1}{\alpha_2 d_2} \right),$$

$$K_2 = \pi / \left(\frac{1}{\alpha_3 d_3} + \frac{1}{2\lambda_2} \ln \frac{d_4}{d_3} + \frac{1}{\alpha_4 d_4} \right),$$

$$K_3 = \pi / \left(\frac{1}{\alpha_1 d_1} + \frac{1}{2\lambda_1} \ln \frac{d_2}{d_1} + \frac{1}{2\lambda_2} \ln \frac{d_4}{d_3} + r_k + \frac{1}{\alpha_4 d_4} \right).$$

We write the boundary conditions for system (1):

$$t_1'(L) = T_1, \quad t_2(0) = T_2, \quad t_1''(L) = T_1. \tag{2}$$

The characteristic equation of system (1)

$$\begin{vmatrix} \dfrac{k_1+k_2}{W_1'} - \gamma & -\dfrac{k_1}{W_1'} & -\dfrac{k_3}{W_1'} \\ \dfrac{k_1}{W_2} & -\left(\dfrac{k_1+k_2}{W_2} + \gamma\right) & \dfrac{k_2}{W_2} \\ -\dfrac{k_3}{W_1''} & -\dfrac{k_2}{W_1''} & \dfrac{k_2+k_3}{W_1''} - \gamma \end{vmatrix} = 0,$$

transformed to the form $\gamma^3 - \alpha\gamma^2 - \beta\gamma = 0$, has the following roots:

$$\gamma_1 = 0, \quad \gamma_{2,3} = \frac{\alpha}{2} \pm \frac{1}{2}\sqrt{\alpha^2 + 4\beta}, \tag{3}$$

where

$$\alpha = \frac{k_2 W_1'(W_2 - W_1'') + k_3 W_2 (W_1'' + W_1') - k_1 W_1''(W_1' - W_2)}{W_1' W_1'' W_2};$$

$$\beta = \frac{(k_1k_2 + k_1k_3 + k_2k_3)(W_1' + W_1'' - W_2)}{W_1' W_1'' W_2}.$$

We write the solution of system (1) in the form

$$\begin{aligned} t_1' &= A_1 + B_1 \exp(\gamma_2 z) + C_1 \exp(\gamma_3 z), \\ t_2 &= A_2 + B_2 \exp(\gamma_2 z) + C_2 \exp(\gamma_3 z), \\ t_1'' &= A_3 + B_3 \exp(\gamma_2 z) + C_3 \exp(\gamma_3 z). \end{aligned} \tag{4}$$

Considering that the response of the sum of the particular solutions of the system of homogeneous differential equations is equal to the sum of their responses, we find

$$A_1 = A_2 = A_3 = A. \tag{5}$$

Inserting (4) and (5) into (1) and considering that the coefficients with $\exp(\gamma_2 z)$ and $\exp(\gamma_3 z)$ in both parts of the equations should be equal, we obtain the following relations between the constants of integration:

$$B_1 = B, \quad B_2 = \xi B, \quad B_3 = \varkappa B, \tag{6}$$

$$C_1 = C, \quad C_2 = \xi^* C, \quad C_3 = \varkappa^* C, \tag{7}$$

where

$$\xi = \frac{k_1(k_2 + k_3 - W_1''\gamma_2) + k_2k_3}{(W_2\gamma_2 + k_1 + k_2)(k_2 + k_3 - W_1''\gamma_2) - k_2^2}; \tag{8}$$

$$\varkappa = \frac{k_3 + k_2\xi}{k_3 + k_2 - W_1''\gamma_2}; \tag{9}$$

$$\xi^* = \frac{k_1(k_2 + k_3 - W_1''\gamma_3) + k_2k_3}{(W_2\gamma_3 + k_1 + k_2)(k_2 + k_3 - W_1''\gamma_3) - k_2^2}; \tag{10}$$

$$\varkappa^* = \frac{k_3 + k_2\xi^*}{k_3 + k_2 - W_1''\gamma_3}. \tag{11}$$

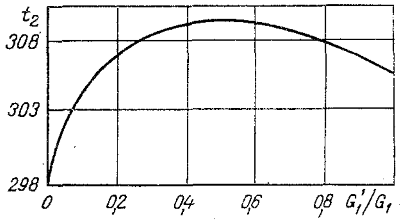

Fig. 3. Dependence of the outlet temperature of the heated medium on G_1'/G_1. t_2, °K.

With allowance for (5)-(11), we write the general solution (4) of system (1) in the form

$$\begin{aligned} t_1' &= A + B\exp(\gamma_2 z) + C\exp(\gamma_3 z), \\ t_2 &= A + \xi B\exp(\gamma_2 z) + \xi^* C\exp(\gamma_3 z), \\ t_1'' &= A + \varkappa B\exp(\gamma_2 z) + \varkappa^* C\exp(\gamma_3 z). \end{aligned} \tag{12}$$

The constants of integration A, B, and C are found from boundary conditions (2). Inserting (2) into (12) and solving the system of equations for A, B, and C, we obtain

$$A = T_1 + \frac{(T_1 - T_2)(\varkappa^* - \varkappa)\exp[(\gamma_2+\gamma_3)L]}{\xi(\varkappa^*-1)\exp(\gamma_3 L) - \xi^*(\varkappa-1)\exp(\gamma_2 L) - (\varkappa^*-\varkappa)\exp[(\gamma_2+\gamma_3)L]}, \tag{13}$$

$$B = -\frac{(T_1 - T_2)(\varkappa^* - 1)\exp(\gamma_3 L)}{\xi(\varkappa^*-1)\exp(\gamma_3 L) - \xi^*(\varkappa-1)\exp(\gamma_2 L) - (\varkappa^*-\varkappa)\exp[(\gamma_2+\gamma_3)L]}, \tag{14}$$

$$C = \frac{(T_1 - T_2)(\varkappa - 1)\exp(\gamma_2 L)}{\xi(\varkappa^*-1)\exp(\gamma_3 L) - \xi^*(\varkappa-1)\exp(\gamma_2 L) - (\varkappa^*-\varkappa)\exp[(\gamma_2+\gamma_3)L]}. \tag{15}$$

Thus, considering (13)-(15), the equations describing the temperature distribution along the heat exchanger can be written in the final form

$$t_1'(z) = T_1 - (T_1 - T_2)\frac{(\varkappa^*-1)\exp(\gamma_3 L + \gamma_2 z) - (\varkappa-1)\exp(\gamma_2 L + \gamma_3 z) - (\varkappa^*-\varkappa)\exp[(\gamma_2+\gamma_3)L]}{\xi(\varkappa^*-1)\exp(\gamma_3 L) - \xi^*(\varkappa-1)\exp(\gamma_2 L) - (\varkappa^*-\varkappa)\exp[(\gamma_2+\gamma_3)L]},$$

$$t_2(z) = T_1 - (T_1 - T_2)\frac{\xi(\varkappa^*-1)\exp(\gamma_3 L + \gamma_2 z) - \xi^*(\varkappa-1)\exp(\gamma_2 L + \gamma_3 z) - (\varkappa^*-\varkappa)\exp[(\gamma_2+\gamma_3)L]}{\xi(\varkappa^*-1)\exp(\gamma_3 L) - \xi^*(\varkappa-1)\exp(\gamma_2 L) - (\varkappa^*-\varkappa)\exp[(\gamma_2+\gamma_3)L]},$$

$$t_1''(z) = T_1 - (T_1 - T_2)\frac{\varkappa(\varkappa^*-1)\exp(\gamma_3 L + \gamma_2 z) - \varkappa^*(\varkappa-1)\exp(\gamma_2 L + \gamma_3 z) - (\varkappa^*-\varkappa)\exp[(\gamma_2+\gamma_3)L]}{\xi(\varkappa^*-1)\exp(\gamma_3 L) - \xi^*(\varkappa-1)\exp(\gamma_2 L) - (\varkappa^*-\varkappa)\exp[(\gamma_2+\gamma_3)L]}.$$

As an illustration, Fig. 2 shows theoretical and experimental temperature profiles along a heat exchanger 0.7 m high with the following geometric and regime characteristics: d_1 = 0.006 m, d_2 = 0.01 m, d_3 = 0.013 m, d_4 = 0.018 m, pitch of the spiralling on the inner tube S = 0.04 m, f_1 = 0.18, f_2 = 0.14 and G_1' = 0.042 kg/sec, G_1'' = 0.041 kg/sec, G_2 = 0.09 kg/sec.

The experimental data were obtained on a unit representing a hydrodynamically closed loop operating on water. The tests were conducted with the following parameters for the investigated single-phase flow: pressure at the channel inlet 0.4-1.7 MPa, temperature 300-403°K. We calculated the optimum ratio of the flow rates of the heat carrier in the inner tube and the space between tubes (Fig. 3). The optimality criterion was the maximum outlet temperature of the heated medium. It follows from the results obtained that the most efficient operating regime for the heat exchanger lies within the range of G_1'/G_1 from 0.47 to 0.51.

Thus, the analytical relations obtained for the temperature profiles agree well with the experimental data and can be used to calculate the temperature fields of heat carriers in high-efficiency heat exchangers with a spiralled inner tube and two-sided heating.

NOTATION

W, water equivalent; t, temperature, °K; z, spatial coordinate, m; α_1, α_2, coefficients of heat transfer from inner and outer surfaces of the inner tube, $W/m^2\cdot K$; α_3, α_4, coefficients of heat transfer from inner and outer surfaces of the outer tube, $W/m^2\cdot K$; p_k, length of arc of contact between spiralled inner tube and outer tube in the channel cross section, m; r_k, contact linear heat-transfer resistance, $m\cdot K/W$; λ_1, λ_2, thermal conductivities of inner and outer tubes, $W/m\cdot K$; T_1, T_2, inlet temperatures of flows of heating and heated media; d_1, d_2, inside and outside diameters of the inner tube, m; d_3, d_4, inside and outside diameters of the outer tube, m; L, length of the heat exchanger, m; G, mass flow rate, kg/sec. Indices used with W, T, and G: 1, heating medium; 2, heated medium; ', medium in inner tube; ", medium in space between tubes.

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CALCULATING THE EXTERNAL HEAT EXCHANGE AND AERODYNAMIC DRAG OF PLATE-FINNED AIR COOLERS

M. N. Chepurnoi, V. N. Lomakin,
V. É. Shnaider, and V. M. Chepurnoi

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Results are presented from an experimental study of heat exchange and pressure loss in plate-finned air coolers with different spacings of the fins.

In practice, mainly two types of air coolers and air condensers are used: those equipped with staggered bundles of tubes with circular or spiral fins, and those equipped with corridor bundles of tubes with plate fins. The hydrodynamics and heat exchange of apparatus of the first type have been studied fairly thoroughly, and recommendations have been made for calculating their thermal and aerodynamic performance characteristics [1, 2]. For apparatus of the second type, until now there has not been adopted a single, proven method of design. This is because, as was noted in [1], their surfaces are not similar. Different investigators [3-6, et al.] have obtained a large number of formulas for calculating heat exchange and friction loss which are suitable only for air coolers of the designs in question.

The use of special methods to analyze test data makes it hard to compare the results of different studies, as is indicated by the surveys in [1, 2]. The use of different linear dimensions as the determining dimensions in similitude criteria as well as the use of different dimensionless simplexes characterizing the geometric similarity of the heat-exchange surfaces, on the one hand, does not permit generalization of the available empirical data and, on the other hand, leads to a significant discrepancy between results obtained under identical test conditions.

In connection with this, we have attempted to refine the laws of external heat exchange and aerodynamic drag for plate-finned air coolers on the basis of studies of models with different fin spacings. We used a 400 × 400 mm wind tunnel to study air coolers with tubes with an outside diameter of 25 mm placed in a corridor arrangement. The tubes were made up of sections 170 mm long having two lengthwise (in the direction of the air flow) and five

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